CSABA HEGEDŰS

RISK-BASED CONSIDERATION OF MEASUREMENT UNCERTAINTY IN DECISIONS

The aim of this study is the revision of conformity and process control techniques and the development of a new method for the design of control procedures and charts. The novelty of the proposed method comes from taking measurement uncertainty and decision risk into account in order to fit the appropriate control procedure to the observed process.

Introduction

Investigation, estimation and handling of risks became necessary tasks of business processes. Risk evaluation processes arise as new standards in several branches of industry or become part of existing ones; for instance, the FMEA in the automotive or micro electric area, the HAZOP in the chemical industry, the HACCP for the foods and PSA/PRA in nuclear power plants (Kovács & Pató Gáborné Szűcs, 2006). However, the treatment of risks is not applied in conformity assessment and control, these methods work only on the reliability base. Decisions based only on the confidence interval and probability of errors lag behind methods employing the consideration of consequences into the decision criteria, too. To minimize the risks a new quality/conformity control approach is proposed with the evaluation of the measurement and sampling uncertainty and modification of decision rules.

Most of the conformity or process controlling decisions are based on measurement results. However, these measurement results have uncertainty that can induce decision errors. Conformity assessment and evaluation of measurement uncertainty are separately handled tasks in everyday practice. In most cases the estimation of measurement uncertainty is only used to choose the adequate measuring equipment and method for a particular assessment job. Previous researches (e.g. Carbone et al., 2003; Ellison & Williams, 2007) treat this problem only from metrology aspects. In this study measurement uncertainty is considered as part of a risk based decision problem in the cases of continuous conformity control, sampling for process control, and forecasting.

Measurement uncertainty

The International Organization for Standardization (ISO) issued a guide (BIPM, et al., 1993) 20 years ago that defines measurement uncertainty as a "parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand" (BIPM, et al., 1993, p. 3). Measurement uncertainty is characterized with the σ_m standard deviation of measured values (and called standard uncertainty or combined standard uncertainty) or with an interval that has a $k\sigma_m$ radius (and called expanded uncertainty) according to the Guide to the Expression of Uncertainty in Measurement (GUM). In the last 20 years, since the first issue of GUM expanded uncertainty came into general use in the practice of measuring laboratories, but measurement uncertainty handling methods have not reached a wide audience outside these laboratories.

The guides and industrial standards – (CENELEC, 1997; Ellison & Williams, 2007; IEC CISPR, 1997; ILAC, 2009; ISO, 1998) – specify the coverage factor values on a reliability base and the length of the interval is often calculated with k=2 coverage factor value. The normal probability distribution is also assumed for the dispersion of the measurement results. The expanded uncertainty interval contains more than 95% of the observations if the distribution is Gaussian, however in any other case the confidence level of the decision is under or over-estimated (Vilbaste et al., 2010). Therefore decisions should consider the whole probability distribution of measurement uncertainty instead of standard deviation or its multiplication with the coverage factor k (Rossi & Crenna, 2006). Transition from the reliability centred conceptualization to the risk based approach is also required.

The decision risk arising from the measurement uncertainty can be mitigated in two ways, with the reduction of the uncertainty (Kosztyán et al., 2010) or taking this uncertainty into account and modifying the decision rules.

Following the GUM many studies (Carbone et al., 2003; Pendrill, 2006; Cox et al., 2008) discussed this problem from a metrological aspect, focusing on the measuring instrument and calibration, and some dealt with the conformity decisions based on the measurement results (Forbes, 2006; Pendrill, 2008). Beges et al. (2010) specified the range of target uncertainty that minimize the total cost from inspection and decision errors. The focus of that research is on the selection of an appropriate measurement method and instrumentation. There was no solution to implement the measurement uncertainty handling in statistical control chart applications.

Consideration of measurement uncertainty in conformance assessment

This section deals with three main area of the conformity control. The first subsection introduces the consideration of measurement uncertainty and decision risk in complete conformity control when all the products are inspected. This method is extended in the second subsection to support the statistical process control, when decisions are based on sampling. To further enhance the consideration of uncertainties and risks in the third subsection the inherent relationship of the consecutive samples are taken into account in order to forecast the next values.

Revision of complete conformity control

In conformity control the measured value is compared to one or two acceptance limits. If the measured value is within the acceptance region the product considered conforming, if it is outside the region the product considered non-conforming and will be rejected. The acceptance limits can be some technical specification limits or stricter control limits. Because of the measurement uncertainty the measurement result y and the actual value of the observed characteristic x differ from each other.

The decision on the conformity of a product based on $y=x+m$ measured value as a sum of the real value and measurement error m but the conformity of the product is influenced by the relation of the actual value x to the upper (USL) and lower (LSL) specification limits. This twoness results in (at least) four different outcomes of the decision (Table 1): correct acceptance, correct rejection and two types of decision errors. In case of decision error type I the actually conforming product is considered non-conforming based on the measured values and rejected superfluously. Decision error type II is made when the measurement uncertainty conceals the non-conformity and the process revision or product rejection fails.

To each outcome r_{ij} proportional revenues and c_{ij} proportional costs are assigned and π_{ij} proportional profits of the decision outcomes are calculated as their difference (r_{ij}-c_{ij}).

		Decision	
		Accept (*j*=1)	**Reject (*j*=0)**
Fact	**Conforming (*i*=1)**	$\pi_{11}=r_{11} - c_{11}$ Correct acceptance	$\pi_{10}=r_{10} - c_{10}$ Superfluous rejection
	Non-conforming (*i*=0)	$\pi_{01}=r_{01} - c_{01}$ Incorrect acceptance	$\pi_{00}=r_{00} - c_{00}$ Correct rejection

Table 1. Particular profits of the four decision outcomes
Source: (Hegedűs, 2014a)

To maximize the expected profit the specification limits, which work as acceptance limits in case of total inspection, are modified with K_L and K_U correction components. The measured value y is compared to the new LSL+K_L lower and USL-K_U upper acceptance limits. If the measured value is between the new limits (LSL+$K_L \leq y \leq$ USL-K_U) the product is accepted otherwise rejected (Figure 1). This approach allows us to define different intervals for each limit to handle asymmetric distributions. Correction component values are determined to minimize the expected profit depending on them.

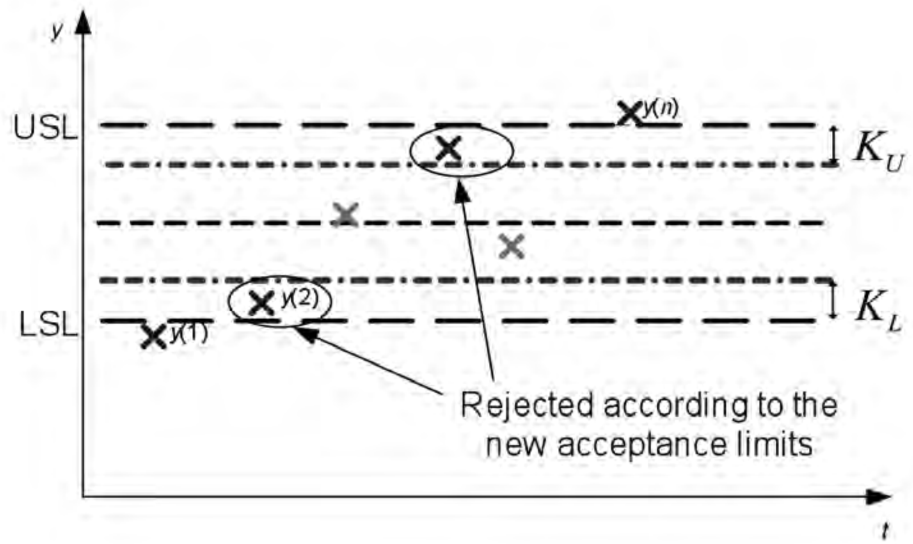

Figure 1. Risk-based modification of acceptance limits
Source: (Hegedűs, 2014b)

Let $\Pi(K_L,K_U)=p_{11}(K_L,K_U)\pi_{11}+p_{10}(K_L,K_U)\pi_{10}+p_{00}(K_L,K_U)\pi_{00}+p_{01}(K_L,K_U)\pi_{01}$ be the expected profit of the decision as a function of K_L and K_U, where $p_{ij}(K_L,K_U)$ is the probability of the outcome *ij* affected by the correction components.

To calculate the probability of outcomes the following Process – Uncertainty of measurement diagram (PU-diagram) have been created (Figure 2). The left side of Figure 2 depicts the four outcomes of Table 1, the black parallelogram in the middle represents the correct acceptance (the values of *x* and *y* are within the specified limits). The cases of decision error type II are presented on the left and the right of this parallelogram (LSL≤*y*≤USL and *x*≤LSL or USL≤*x*). Above and below the parallelogram the cases of decision error type I are presented. The unmarked fields belong to the fourth case, the case of correct (required and performed) revision or control. The probability of an outcome is calculated as the volume bounded by the corresponding area on the diagram and the two dimensional probability density function above it.

If K_L and K_U correction factors are positive the acceptance zone is tightened, the negative value of the correction factors means the relaxation of the acceptance region. By weighting the profits (or loss) of decision outcomes with the value of occurrence probability, the expected profit maximizing objective function can be formulated as the following equation:

$$\Pi(K_L,K_U)=\Pi(0,0)+\Delta\Pi(K_L,K_U)\rightarrow \max \tag{1}$$

The $\Pi(0,0)$ is the expected profit without correction ($K_L=K_U=0$), $\Delta\Pi(K_L,K_U)$ is the alteration of the expected profit as a function of K_L and K_U. It is sufficient to maximize $\Delta\Pi(K_L,K_U)$ profit alteration in order to maximize $\Pi(K_L,K_U)$ expected profit.

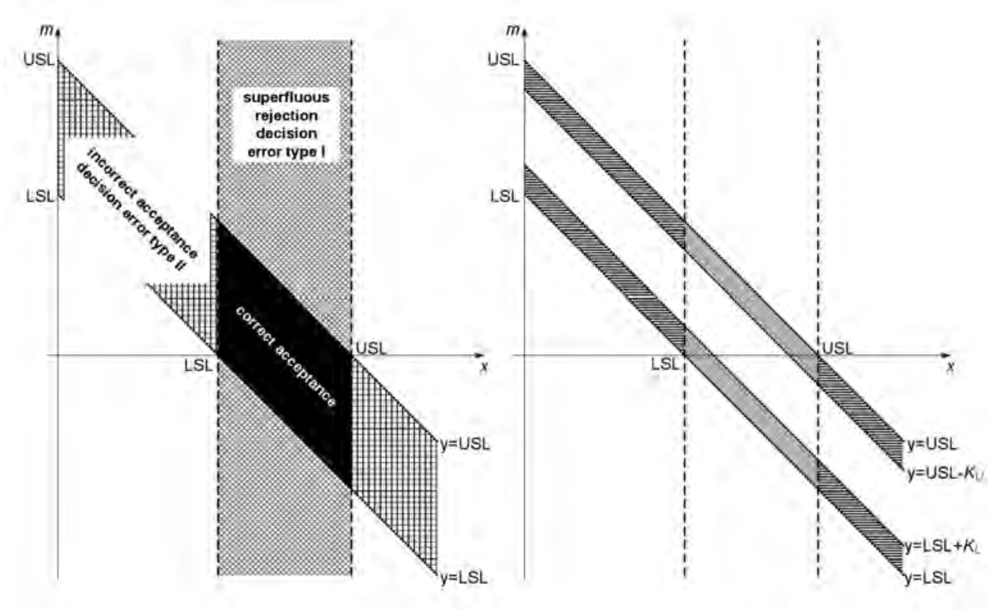

Figure 2. The regions of the four cases of decision outcomes (on the left side) and the regions affected by the alteration of acceptance limits
Source: (Hegedűs, 2014a)

In some simple cases the optimal values of correction components can be determined analytically, in the others numerical methods or simulations are required.

Simulations have been carried out to compare the typical cases (Table 2): when the measurement uncertainty is not taken into account (K=0), the acceptance region is tightened ($K=2\sigma_m$) or relaxed ($K= -2\sigma_m$) with the interval that the industrial standards suggest. Let the q denote the ratio between the proportional losses of the decision errors calculated with the following formula:

$$q = \frac{\pi_{11} - \pi_{10}}{(\pi_{11} - \pi_{10}) + (\pi_{00} - \pi_{01})} \quad q \in \left]0,1\right[\qquad (2)$$

The q=0.5 indicates the equality of proportional losses of the two kinds of decision errors if the cost of decision errors are compared to the cost/profit of correct decision. The higher values of q show that the proportional loss of a decision error type I is higher than the proportional loss of the other kind of decision error. The lower values indicate the dominancy of proportional loss from decision error type II.

In Table 2 the proportional profits (or losses) associated with these cases are compared to the proportional profit gained with the optimal correction ($K=K_{opt}$). In the first three columns are the profits/losses connected to the typical cases. The highest value from these three is marked with bold and underlined type-face. This shows which typical solution is worth to use for each value of q in this illustrative setting [$x \sim N(\mu_x=105, \sigma_x=4)$, $m \sim N(\mu_m=0, \sigma_m=2)$, LSL=100]. According to these results the "guard band" calculated only from the (combined) standard uncertainty is not the best solution for every cases, its width varies with change of q. The maximal profit is in the fourth column and the corresponding optimal extent of correction is shown in the last column. Consequently the value of K_{opt} is not only dependent on the measurement uncertainty the costs and revenues of the decision outcomes should also be taken into account.

q	Proportional profit/loss				K_{opt}
	K=-2σ_m	K=0	K=2σ_m	K=K_{opt}	
0,05	-10,5472	2,5184	**4,9113**	5,6933	2,4280
0,10	-0,4061	5,4685	**4,9865**	6,3438	1,6156
0,15	2,9743	**6,4519**	5,0115	6,7386	1,0675
0,20	4,6644	**6,9436**	5,0240	7,0247	0,6319
0,25	5,6785	**7,2386**	5,0316	7,2500	0,2582
0,30	6,3546	**7,4353**	5,0366	7,4362	-0,0774
0,35	6,8375	**7,5758**	5,0401	7,5951	-0,3884
0,40	7,1997	**7,6812**	5,0428	7,7339	-0,6835
0,45	7,4814	**7,7631**	5,0449	7,8572	-0,9690
0,50	7,7068	**7,8287**	5,0466	7,9683	-1,2500
0,55	**7,8911**	7,8823	5,0480	8,0695	-1,5310
0,60	**8,0448**	7,9270	5,0491	8,1625	-1,8165
0,65	**8,1748**	7,9648	5,0501	8,2487	-2,1116
0,70	**8,2862**	7,9972	5,0509	8,3291	-2,4226
0,75	**8,3828**	8,0253	5,0516	8,4046	-2,7582
0,80	**8,4673**	8,0499	5,0522	8,4758	-3,1319
0,85	**8,5419**	8,0716	5,0528	8,5435	-3,5675
0,90	**8,6082**	8,0909	5,0533	8,6083	-4,1156
0,95	**8,6675**	8,1081	5,0537	8,6707	-4,9280

Table 2. The proportional profit or loss as a function of K and q
(μ_x=105, σ_x=4, μ_m=0, σ_m=2, LSL=100)
Source: (Hegedűs, 2014a)

To inspect the relationship between the total profit and the relative distance of the process mean from the acceptance limit a new simulation have been done. On Figure 3 the original total profit $\Pi(0)$ is represented by the grey surface and the total profit associated with the acceptance limit modification $\Pi(K)$ is depicted as black surface. The white dashed line belongs to $\Pi(K=-2\sigma_m)$, the total profit from the relaxation of the acceptance region according to the industrial standards. The solid white line marks the maximal total profit that results from the optimal correction of acceptance limit.

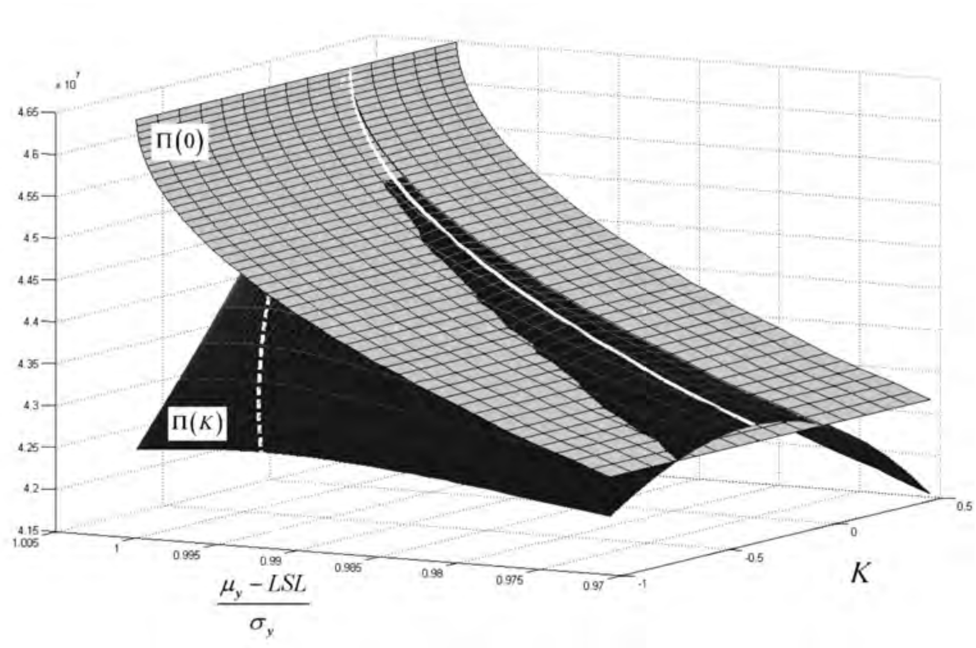

Figure 3. The profit depending on the correction component [$\Pi(K)$] compared to the original profit [$\Pi(0)$] as functions of relative distance of the process mean from the lower bound
Source: (Hegedűs & Kosztyán, 2011)

Consideration of uncertainties in the use of variable control charts

If the conformity of a population is determined in accordance with the results estimated from a sample, e.g. in acceptance sampling and statistical process control (SPC), the uncertainty of estimation also has to be considered beside the measurement uncertainty.

On the control charts of the statistical process control (SPC) the measured values are compared to calculated control limits instead of the specification limits. Falling outside these control limits does not mean the non-conformity of the product (or process) just indicate the necessity of process control or revision. In the SPC the process is said to be in control if the probability of next values falling into a given interval can be determined on the basis of previous observations (Shewhart, 1931) (Montgomery, 1996). The use and analysis of control charts is practically a hypothesis test (Neyman & Pearson, 1933), the null hypothesis assumes that the current process is the same (has the same probability distribution and parameters) as the previously investigated and considered in control.

The probability distribution of the observed value is assumed to be normal (Gaussian) for the specification of the control chart limits, but this assumption is not true in every case (Schippers, 1998). If the sample size falls below 4 or an individual value chart is applied, the non-normality could increase the decision errors significantly (particularly type I errors) (Schilling & Nelson, 1976).

If there is an asymmetry in the probability density function of the observed variable a greater skewness can cause significant bias from the normality of the sample mean even with sample size of 5-10, according to the Berry–Esseen inequality (Esseen, 1956) and calculation of Shevtsova (2011). Similarly the assumption of normal distribution is incorrect and misleading when the expected value of the observed characteristic is near to zero but the set of negative values is not part of the domain, e.g. measuring weight or concentration.

The deviation from the assumed ratio of decision errors causes great problems if one of the decision errors has significantly more severe consequences than the other. This difference of the decision error consequences is not reflected in the rules that are assuming Gaussian distribution of the observed characteristic and based on a reliability centred approach.

According to Albers et al. (2006) if the observed values cannot be considered to follow normal distribution then not the distribution that has the best value on the standard goodness of fit test will necessarily be the best for the calculation or simulation. The standard goodness of fit test concentrates on the middle of the distribution (the surrounding of mode and/or median) not on the tails. Since the majority of the points fall near to the middle of the distribution these points will determine the "goodness of fit". The conformity control deals with the instances that are near the bound or outside of the acceptance region and these instances are typically on the tails of the distributions. Therefore a new test of fit or a new objection function that takes the consequences of the decision is required.

Optimal modification of control limits

Let the probability density function of x real values be $f(x)$, and the probability density function of m measurement error be $g(m)$. These two distributions are assumed to be independent of each other, and the common distribution is calculated as their multiplication.

Simulations have been run to determine the optimal alteration of control limits. The probability distribution of measured values (and its parameters) can be determined from the initial inspections. The measurement uncertainty can be obtained from the calibrations, previous experience and the documentation of measuring system analyses. The x real values are estimated as the difference of y measured values and m measurement error. The probability distribution of x real values (and its parameters) comes from the deconvolution of distribution of y measured values using the knowledge about the probability distribution (and its parameters) of measurement uncertainty.

During the simulation the same structure of proportional profits can be used with a minor change in the interpretation: the non-conformance of the process not means the non-conformance of the product but the necessity of control actions, therefore the costs are associated with these control actions not with the rejection of the product. The $\Sigma\Pi$ total profit in reference to decisions can be calculated in the simulation:

$$\sum \Pi = q_{11} \cdot \pi_{11} + q_{10} \cdot \pi_{10} + q_{01} \cdot \pi_{01} + q_{00} \cdot \pi_{00} \tag{3}$$

The q_{ij} number of elements belongs to certain cases calculated in the simulation. To maximize the expected profit let the decision rules be modified. The value of correction factors are calculated in the simulation. These correction factors are not coefficients; they give directly the extent of the alteration of specification limits. If the risk of decision error type II is low the value of correction factor can be negative. In this case the control limits do not become stricter rather wider. The Monte Carlo simulation searches the value of K_L and K_U that determine the maximum of total profit in reference to decisions.

$$\begin{aligned}\Sigma \Pi(K_L, K_U) = & q_{11}(K_L, K_U) \cdot \pi_{11} + q_{10}(K_L, K_U) \cdot \pi_{10} + \\ & + q_{01}(K_L, K_U) \cdot \pi_{01} + q_{00}(K_L, K_U) \cdot \pi_{00} \rightarrow \max\end{aligned} \tag{4}$$

Practical example to the modification of SPC charts

At a supplier in the automotive industry the housing of the fuel pump is manufactured by injection moulding. The diameter of the flanged top cover of this housing is a critical parameter because it affects the ability to assemble the fuel delivery system into the tank. To assure the proper nesting and sealing the diameter must be 121 millimetres and the maximal deviation from this target value should not exceed 0.2 millimetres. The conformity of this parameter is controlled with x-bar (sample mean) chart and sample size of 3.

The observed value follows a Weibull distribution with α= 121.018 scale parameter and β=1,659.907 shape parameter. The measurement uncertainty can be described with normal distribution with μ_m= 0 mean and σ_m= 0.038 standard deviation.

In the following steps of the manufacturing the flanged top is welded to the other part of the housing that contains the fuel pump. If a non-conforming flanged top is accepted and assembled additional costs appear from the destructive disassembling of the housing and regaining of the fuel pump. Since the capability of the process is low (c_{pk}= 0.704) these extra costs are calculated in addition to the manufacturing cost (π_{01}= −19.31) if decision error type II is committed and the necessary control fails to be carried out. If the control chart incorrectly shows that the process is out of control we face with the π_{10}= −1.492 loss. The two correct decisions come with π_{00}= −1.864 loss and π_{11}=0.372 profit.

The use of the x-bar chart have been investigated in case of three different sample sizes – 3, 5 and 7 – and compared to the Gaussian distribution with the same mean and standard deviation (Table 3). In all the six cases the initial control limits are defined according to the general chart design rules ($\pm 3\sigma$ from the centre line). The correction components tightened the acceptance interval in every case, and the extent of the modification not exceeds the σ_m standard deviation that describes the measurement uncertainty.

Probability distribution	Sample size	LCL	$K_{L,opt}$	LCL_{opt}	UCL	$K_{U,opt}$	UCL_{opt}
Weibull	**3**	120.8751	0.0125	120.8876	121.0769	0.0225	121.0544
	5	120.9155	0.0175	120.9330	121.9155	0.0175	121.898
	7	120.9328	0.015	120.9478	121.0192	0.015	121.0042
Gaussian	**3**	120.8867	0.02	120.9067	121.0733	0.02	121.0533
	5	120.924	0.0175	120.9415	121.036	0.0175	121.0185
	7	120.94	0.015	120.9550	121.0199	0.15	120.8699

Table 3. Control limits for variables following Weibull and Gaussian distribution
Source: (Hegedűs, 2014a)

If the sample size is small we get different optimal upper ($K_{U,opt}$) and lower ($K_{L,opt}$) correction components for the two distributions (Table 3) because of the differences on the tails of these distributions (Figure 4).

The skewness of the Weibull distribution of the observed parameter is -0.9043 (Figure 5). If the sample mean is investigated the skewness decreases with the increase of the sample size, it equals to -0.5218 for the chart points calculated from three value (*n*=3), it is -0.4029 for *n*=5 and -0.3414 for samples with seven elements (see Figure 6).

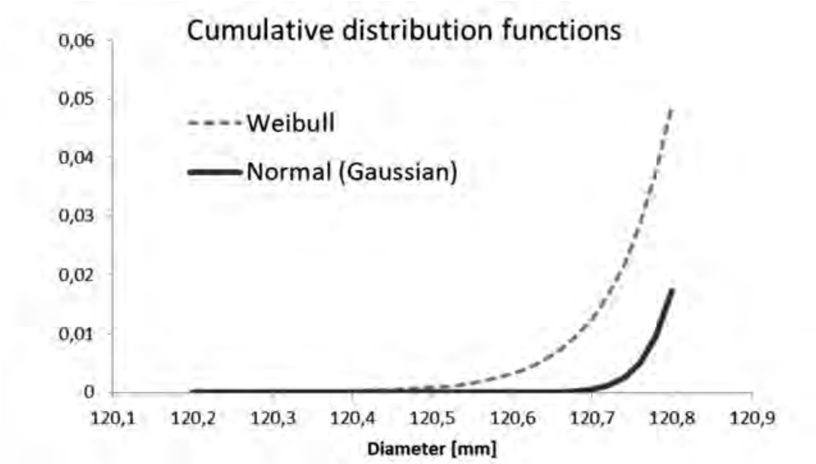

Figure 4. The left tails of the cumulative distribution functions

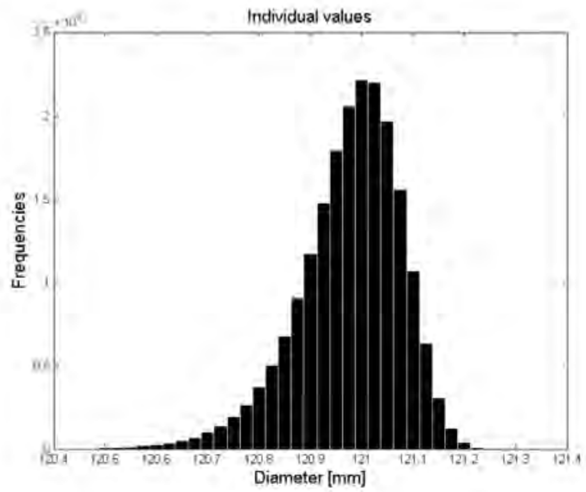

Figure 5. The skewness of the Weibull (α=121.018, β=1659.907) distribution
Source: (Hegedűs, 2014a)

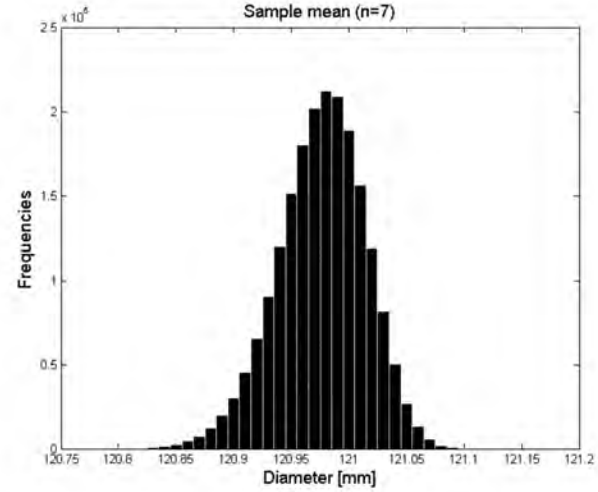

Figure 6. The skewness for samples of seven elements
Source: (Hegedűs, 2014a)

Two million samples in Monte Carlo simulation are not enough to detect significant difference in the sample means outside the control limits. Therefore in this example normal distribution can be used instead of Weibull if just ppm (defective part per million) quality level required and the sample size is at least five.

Since the loss of decision error type II is much higher than the loss of decision error type I the correction of the control limits spares the 36.79 percent of the control cost. This spared control cost is 0.856 percent of the total manufacturing cost of the product and ensue from the change of the modification of decision outcome probabilities.

The investigated process is in the in-control-state in the 83 percent of the time and handled by the x-bar chart (n=5) as statistically controlled in 77.99 percent of the two million cases (Table 4). That means the ratio of decision error type I is five percent and the ratio of decision error type II is also high (4.7 percent).

To find the balance of the four cases that provide the maximal profit or minimal cost the control decisions must be modified by the alteration of the control chart limits. Monte Carlo simulations have been carried out to specify the optimal value of K_L and K_U correction components.

	Acceptance cases	Acceptance percent	Revision cases	Revision percent	Sum cases	Sum percent
In-control	1,559,895	77.99%	100,028	5.00%	1,659,923	83.00%
Out-of-control	93,937	4.70%	246,140	12.31%	340,077	17.00%
Sum	1,653,832	82.69%	346,168	17.31%	2,000,000	100.00%

Table 4. Initial cases of the conformity control of flanged top
Source: (Hegedűs, 2014a)

The alteration of control chart limits modifies only the decisions thus the sum of each column. Since leaving the process in a state of out-of-control results the higher cost the optimisation going to decrease the ratio of these instances.

	Acceptance cases	Acceptance percent	Revision cases	Revision percent	Sum cases	Sum percent
In-control	1,295,340	64.77%	364,583	18.23%	1,659,923	83,00%
Out-of-control	23,160	1.16%	316,917	15.85%	340,077	17,00%
Sum	1,653,832	65.93%	346,168	34.08%	2,000,000	100.00%

Table 5. Ratio of the cases after the modification of control chart
Source: (Hegedűs, 2014a)

Due to the optimisation the number and ratio of process revisions will grow from 17.31 percent to 34.08 percent. The consequence of the more frequent revision is the increase in the ratio of type I errors but it also decrease the ratio of decision error type II with more than 75 percent. Thus the control is overdone because of the low process capability.

Considering uncertainties and risks in forecasting

To improve the conformity control we can take advantage of the inherent relationship of the consecutive measurement results. If this relationship can be described by a stochastic process the next values can be predicted. With the previously introduced correction of acceptance limits and the confidence intervals of the prediction the next time when measurement needs to be performed can be determined at a given level of risk (Figure 7).

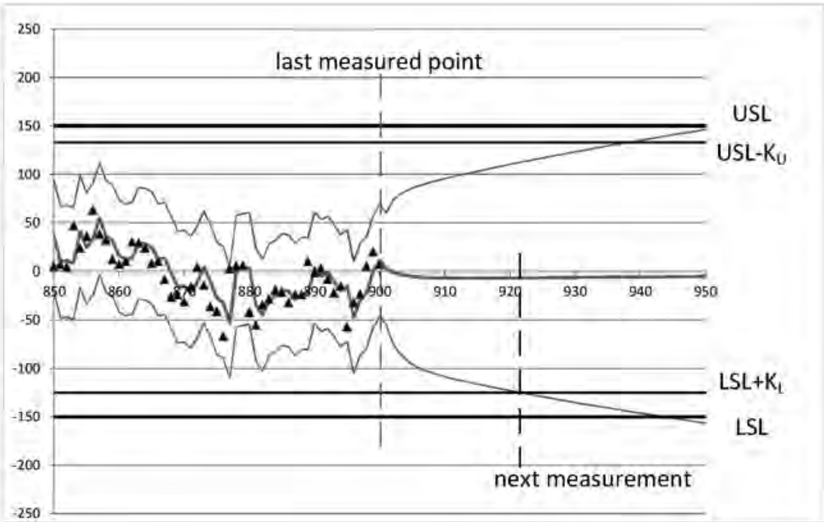

Figure 7. Forecasting between the modified acceptance limits with stochastic processes
Source: (Hegedűs, 2014a)

More suitable to handle the process as a time series particularly in maintenance decisions if deterioration occurs and the process of the observed characteristic of the device has a trend. In order to treat the time series of the observed characteristic with linear stochastic models the time series must be decomposed. The trend shows the expected value of the characteristic. The uncertainty of this forecast derived from the random variation of real value, the frequency of sampling, the sample size and the time interval of the forecast. If the intervals between the samplings are equal the width of the confidence interval of the trend is constant. The lower and upper bound of confidence interval parallel to the trend (Figure 8).

At a given confidence level the width of confidence interval can be decreased if the sampling frequency is increased when the trend comes closer to the LSL (Figure 9). Increasing the frequency of sampling, the length of the confidence interval of forecasting will decrease. The length of the confidence interval (for a given significance level α) can be calculated as follows:

$$INT_{1-\alpha} = \bar{y} \pm t_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \cdot \sqrt{1 - \frac{n}{N}} \quad (4)$$

where n is the size of the sample, N is the number of the elements of the whole population, σ is the uncertainty expressed as a standard deviation and t is the value of Student-t distribution that belongs to the confidence level of 1-α/2.

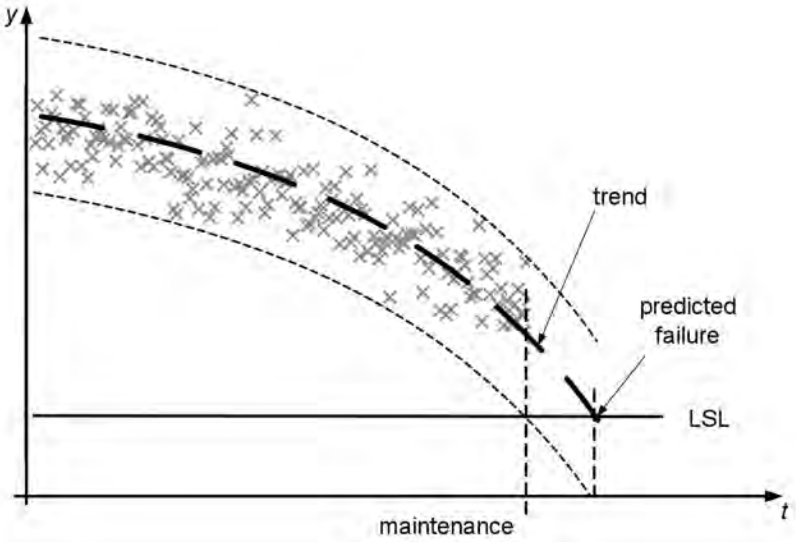

Figure 8. Confidence interval around the measured process
Source: (Hegedűs & Kosztyán, 2011)

After the decomposition we identify the stochastic process best fitting to the real process. Once we have identified a particular model we need to estimate the parameters and assess how well the model fits. After the validation of the stochastic model it can be used for forecasting. This model predicts the next value on the basis of actual and previous values of real process and prediction error. The further we try to forecast the higher the uncertainty will be (see Figure 10).

The optimal control limit can be determined by simulation or estimation with the methods shown in previous sections. This limit is not a constant as it changes with the time. At the time of the initial measurements the risk of decision error type II is low, because the observed characteristic is far from the LSL (see Figure 10). This risk is increasing because of the deterioration. The risk of decision error type I is also increasing but at a slower rate than the risk of decision error type II. So the curve of minimal total risk will increase.

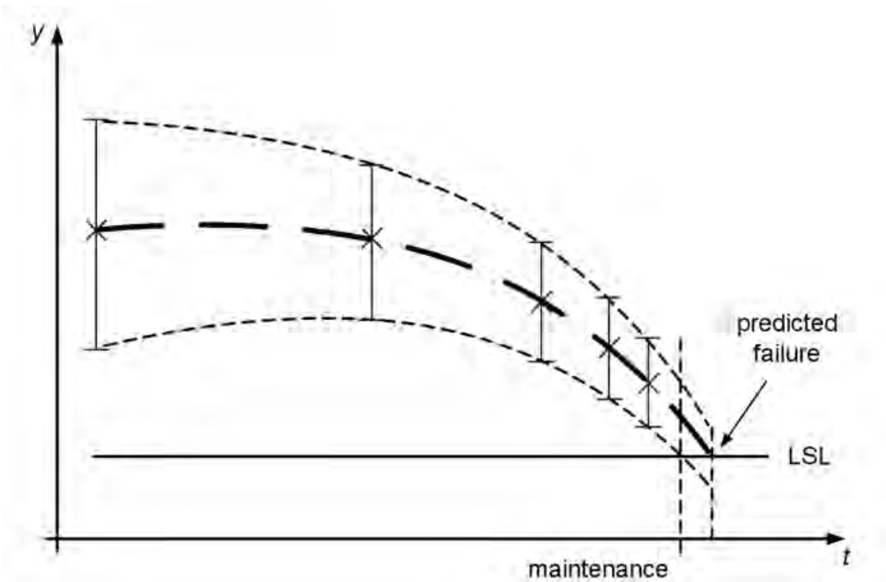

Figure 9. Approximating the level of device failure measurements are taken more frequently to reduce the confidence interval of prediction

Source: (Hegedűs & Kosztyán, 2011)

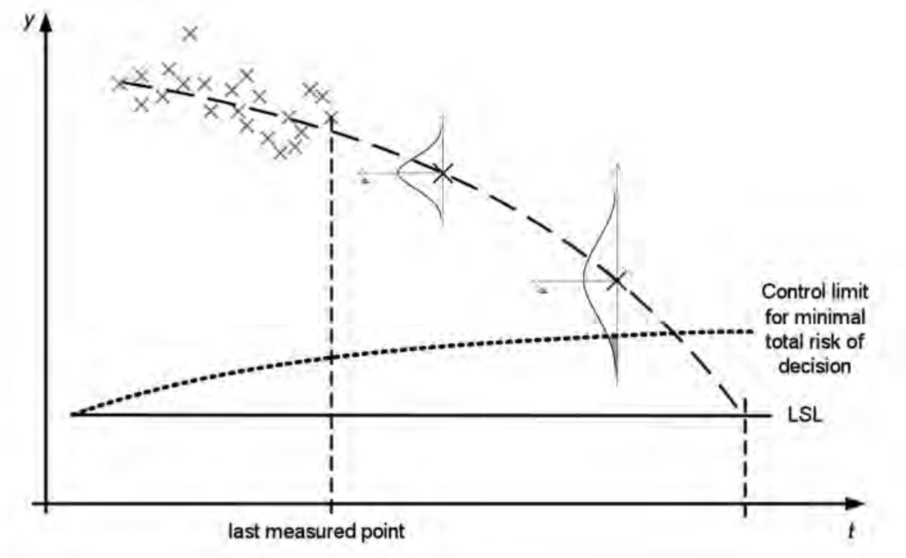

Figure 10. Also the control limits and the confidence interval of the prediction change with the time
Source: (Hegedűs & Kosztyán, 2011)

If the trend of the deterioration and the stochastic model that describes the stationary process are combined the time of the measurements can be determined.

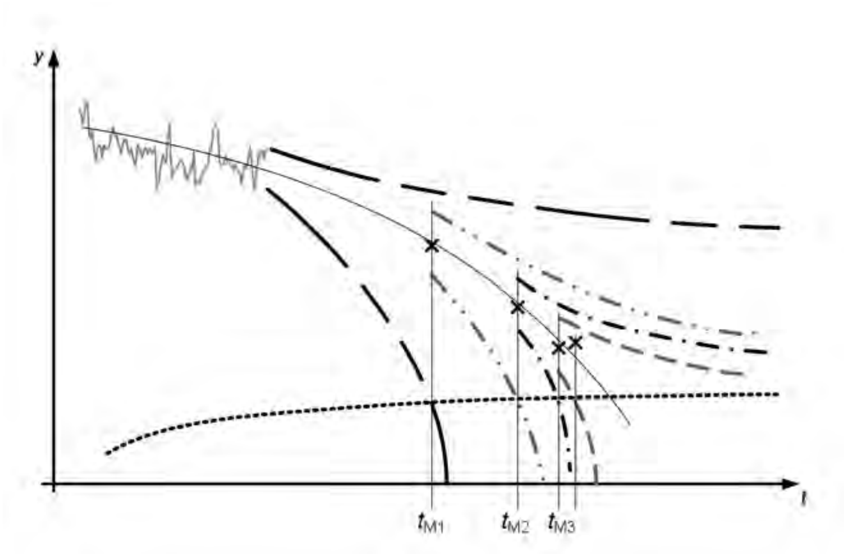

Figure 11. The intersection of prediction confidence intervals and the control limit determine the next time of measurement or maintenance
Source: (Hegedűs & Kosztyán, 2011)

The following measurement must be performed there, where the confidence interval of prediction intersects the curve of minimal total risk (t_{M1}) (see Figure 11). Until this point failure will not occur in the process with the confidence level of forecast. With the new measurement result the decomposition, identification, estimation of parameters and forecast will be executed again. These steps are performed iteratively (t_{M2}, t_{M3}) until the interval between intersection of the confidence interval and the curve of minimal total cost and the intersection of the confidence interval of the trend and the curve of minimal total risk is inessential. At this point maintenance is required as opposed to measurement.

Summary

The industrial conformity assessment or process controlling decisions are simplified to ease the understanding and the everyday work. However, the assumptions and requirements of the used methods are not true for all the case. Due to the computer aided decisions support these simplifications and presuppositions are no longer required and new, more precise methods are available to the managers and operators.

In this paper simulation methods are introduced to reduce the risk from uncertainty of measurement and estimation. These methods no longer require the normality of a process, the deviation from the Gaussian distribution can be handled with the modification of the acceptance limit. The focus of the optimisation is on the decision consequences, the decision rules are modified to decrease the risk of the conformity control. The possible outcomes of a decision are taken into account and costs and revenues are associated with them.

The preventive maintenance decisions are based on measurement results, but these results have an uncertainty and cause incorrect decisions. It is necessary to take into account this uncertainty on a risk base. In this paper a uniform model was presented that treats the customer's risk along with the producer's risk through the consideration of the measurement uncertainty and costs or losses in reference to maintenance decisions. This model gives the optimal control limit of the process that minimizes the total risk associated with the decisions and maximizes the related profits. It can treat both kinds of the processes that have either only one or two specification limits. The optimal control limit influenced by the risks can be determined by Monte Carlo simulation.

The methods introduced here require the user to describe the uncertainty of measurement and the process of the observed characteristics with a probability function, and the cost and revenues should be determined correctly. This means that the managers need to know profoundly the technical and economic attributes of these processes.

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